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TRIALLEL EXPERIMENTS WITH RECIPROCAL EFFECTS

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SUMMARY

The theoretical aspect of triallel analysis has been dealt with by Rawling and Cockerham [7], Hinkelmann [2] and Ponnu Swamy *et al.* [4]. It was shown by Zelen [9] that a partially balanced design with $(m+1)$ associate classes can be derived from a partially balanced design with m associate classes by replacing each treatment by n treatments. In the present study, a four class association scheme has been derived from a partially balanced design with 3-associate classes. The total $(v-1)$, i.e. $[p(p-1)(p-2)-1]$ degrees of freedom (d.f.) has been partitioned into 4 sets of $(p-1, p(p-3)/2, p(p-1)(p-5)/6$ and $5p(p-1)(p-2)/6$ d.f, said to belong to general combining ability (g.c.a.) effects, first order specific combining ability (s.c.a.) effects, second order s.c.a. effects and reciprocal effects respectively. There is complete balance over these sets of degrees of freedom, in the sense of Shah [8].

Keywords : Partially Balanced Design, General and specific combining ability, reciprocal effect, Orthonormal basis.

Introduction

Each individual cross in a diallel experiment is the product of two parents. On the contrary, a three-way cross or a triallel is a product of three parents, for instance $(AABB)\ CC$. Taking p as the number of parents, all possible three way crosses would be $6pC_3$ i.e. $p(p-1)(p-2)$. The theoretical aspect of triallel analysis has been dealt with by Rawling and Cockerham [7], Hinkelmann [2], and Ponnuswamy *et. al.* [4] taking

$3 \cdot pc_3$ i.e. $p(p-1)(p-2)/2$ crosses. The three way cross analysis provides additional information regarding epistatic components of variances and the effects of order in which the parents are involved in crosses. The analysis of triallel experiments taking all the $p(p-1)(p-2)$ triallel crosses is given.

2. Confounded Triallel Experiments

Let $v = p(p-1)(p-2)$ crosses of a triallel experiment with reciprocal crosses be denoted by ijk. Let $p(p-1)(p-2)$ crosses of this triallel experiment be represented by the $v = p(p-1)(p-2)$ treatments of an incomplete block design with b blocks, ijkth cross representing the ijkth treatment. Let y_{ijkl}, the yield of the ijkth treatment (cross) allotted to a plot in the lth block be given by

$$Y_{ijkl} = m + t_{ijk} + \beta_l + e_{ijkl}, \quad i \neq j \neq k ; \tag{1}$$

$$(i, j, k = 1, 2, \ldots, p ; \quad l = 1, 2, \ldots, b)$$

where m is the general mean; t_{ijk} is the three line cross effect; β_l is the lth block effect; and e_{ijkl}'s are the random errors which are independently and normally distributed with means 0 and variances σ^2.

Assume m, t_{ijk}'s, β_l's to be fixed unknown parameters with

$$\sum_{i \neq j \neq k} \sum \sum t_{ijk} = 0 \text{ and } \sum_l \beta_l = 0$$

Let $t_{ijk} = g_i + g_j + g_k + s_{ij} + s_{ik} + s_{jk} + s_{ijk}$ with (2)

$$\Sigma g_i = 0; \quad \sum_{j \neq i} s_{ij} = 0 \quad \text{for all } i; \; s_{ij} = s_{ji} ;$$

$$\sum_{k \neq j \neq i} s_{ijk} = 0 \qquad \text{for all } i \text{ and } j;$$

$$s_{ijk} = s_{ikj} = s_{jik} = s_{jki} = s_{kij} = s_{kji}; \tag{3}$$

$$r_{ijk} + r_{ikj} + r_{jik} + r_{jki} + r_{kij} + r_{kji\,kji} = 0$$

Call g_i as the general combining ability (g.c.a.) effect of the ith line, s_{ij} as the first order specific combining ability (s.c.a.) effect of the ith and jth lines, s_{ijk} as the second order s.c.a. effect of the ith, jth and kth lines and r_{ijk} as the reciprocal effect of the ith, jth and kth lines.

Let $\underline{t} = [t_{123}, t_{132}, t_{124}, t_{142}\, m, \ldots t_{1p(p-1)}, t_{1(p-1)p},$

$t_{213}, t_{231}, \ldots, t_{2(p-1)p}, t_{2p(p-1)}, \ldots, t_{p(p-2)(p-1)},$ (4)

$t_{p(p-1)(p-2)}]$; $t_{ij} = \sum_k t_{ijk}, \; (k \neq i, j)$;

$$t_{i..} = \Sigma\, t_{ij.}\,,\ (j \neq i);\ t^{*}_{ijk} = t_{ijk} + t_{ikj}$$

$$+\, t_{jik} + t_{jki} + t_{kij} + t_{kji},$$

with the analogous definitions of $t^{*}_{ij.}$, $t^{*}_{i..}$, Q, $Q_{ij.}$, $Q_{i..}$ and Q^{*}_{ijk} where Q_{ijk} is the adjusted treatment total for the ijkth treatment (cross) and is defined as

$$Q_{ijk} = Y_{ijk.} - (B^{ijk}/k) \tag{5}$$

when $Y_{ijk.}$ is the total of the plots to which the ijkth treatment (cross) was allotted and B^{ijk} is the total of the blocks containing the ijkth treatment (cross).

Then $g_i = t^{*}_{i..}/6(p-2)(p-3);$

$$s_{ij} = [t^{*}_{ij.} - 6\,(p-3)\,(g_i + g_j)\,]/6(p-4)\,; \tag{6}$$

$$s_{ijk} = (t^{*}_{ijk}/6) - g_i - g_j - g_k - s_{ij} - s_{ik} - s_{jk};$$

$$r_{ijk} = t_{ijk} - t^{*}_{ijk}/6.$$

It was shown by Zelen [9] that a partially balanced design with $(m+1)$ associate classes can be derived from a partially balanced design with m-associate classes by replacing each treatment by a set of treatments.

Use this result and introduce an useful four class association scheme for $v = p(p-1)(p-2)$ treatments represented by ijk $(i \neq j \neq k)$; $i, j, k = 1, 2, \ldots, p$.

By multiplying each treatment ijk of the extended triangular $ET(p)$ design of John [3], an useful 4-class Partially Balanced Incomplete Block ($PBIB$) design is obtained for the $p(p-1\,(p-2)$ treatments to be called a modified extended triangular [$MET(p)$] design with the following association scheme.

Definition 2.1. Let each treatment ijk of the Extended Triangular (ET) association scheme of John [3] be mapped on [ijk, ikj, jik, jki, kij and kji]. For a treatment ijk, the treatments ikj, jik, jki, kij, kji are first associates; and images of ith associates of the ET association scheme are $(i+1)$ + th associates $(i = 1, 2, 3)$.

We define

Definition 2.2. A $PBIB$ design with $v = p\,(p-1)\,(p-2)$ treatments having the association scheme given in Definition 2.1 will be called a modified extended triangular (*MET*) design.

Let N be the incidence matrix of a connected $MET(p)$ design. Then the eigen values θ_i of NN' with their multiplicities α_i (see P.W.M. John, [3]) will be

$$\theta_0 = rk, \quad \alpha_0 = 1;$$

$$\theta_1 = r - \lambda_1, \quad \alpha_1 = 5p(p-1)(p-2)/6;$$

$$\theta_2 = r + 5\lambda_1 + 6\,[(2p-9)\lambda_2 + \lambda_3 (p-4)(p-9)/2 - \lambda_4 (p-4)(p-5)/2\,]$$

$$\alpha_2 = (p-1) \tag{7}$$

$$\theta_3 = r + 5\lambda_1 + 6\,[(p-7)\lambda_2 - 2(2p-11)\lambda_3 + (p-5)\lambda_4],$$
$$\alpha_3 = p(p-3)/2;$$

$$\theta_4 = r + 5\lambda_1 + 6(-3\lambda_2 + 3\lambda_3 - \lambda_4),$$
$$\alpha_4 = p(p-1)(p-5)/6$$

The eigen values ϕ_i of the C-matrix of the given $MET(p)$ design with multiplicities α_i will be

$$\phi_i = r - \theta_i/k, \quad i = 0, 1, 2, 3, 4. \tag{8}$$

Let

$$t_{i}^{*}.. = \underline{l}_i' \underline{t}\,;$$

$$t_{ij}^{*}. = \underline{l}_{ij}' \underline{t}\,, \quad (i < j); \tag{9}$$

$$t_{ijk}^{*} = \underline{l}_{ijk}' \underline{t}\,, \quad (i < j < k);$$

$$t_{ijk} = \underline{x}_{ijk}' \underline{t}\,, \quad i \neq j \neq k\,.$$

From (6), it can be seen that $t'x_i$ $(i = 2, 3, \ldots, p)$ represent g.c.a. contrasts; $\underline{t}'y_i$ $(i = 1, 2, \ldots, p(p-3)/2)$ represent the first order s.c.a. contrasts; $\underline{t}'\underline{z}_i$ $(i = 1, 2, \ldots, p(p-1)(p-5)/6)$ represent the second order s.c.a. contrasts and $\underline{t}'\underline{u}_i$ $(i = 1, 2, \ldots, 5p(p-1)(p-2)/6)$ represent the reciprocal contrasts.

Let S_1, S_2, S_3 and S_4 be the vector spaces spanned by $\underline{l}_i$'s, $\underline{l}_{ij}$'s $\underline{l}_{ijk}$'s and $\underline{m}_{ijk}$'s, reprpectively. Then it can be easily seen that S_1 is a sub-spaee of S_1, S_2 is a sub-space of S_3 and S_3 is a sub-space of S_4. Let $E_{m,n}$ be $m \times n$ matrix with all its elements unity. Then

$$[(1/\sqrt{v})\, E_{v,1},\ \underline{x}_2,\ \underline{x}_3,\ \underline{x}_4,\ \ldots,\ \underline{x}_p] \tag{10}$$

is an orthonormal basis for the vector space S_1 where $\underline{x}_i$'s are given by

$$\underline{x}_i'\underline{t} = \left[\sum_{j=1}^{i-1} t_j^{*}.. - (i-1)\, t_i^{*}..\right] + [3i\,(i-1)\,(p-2)\,(p-3)]^{\frac{1}{2}} \tag{11}$$

$$i = 1, 2, 3, \ldots, p$$

Let us, now extend the orthonormal basis given in (10) of S_1 to an orthonormal basis

$$[(1/\sqrt{v})\, E_{v,1},\ \underline{x}_2,\ \underline{x}_3,\ \ldots,\ \underline{x}_p,\ \underline{y}_1,\ \underline{y}_2,\ \ldots,\ \underline{y}_{p(p-3)/2}] \tag{12}$$

for S_2 [cf. Raghavarao (6)]. Let us, further, extend the basis given in (12) to another orthonormal basis.

$$[(1/\sqrt{v})\, E_{v,1},\ \underline{x}_2,\ \ldots,\ \underline{x}_p,\ \underline{y}_1,\ \underline{y}_2,\ \ldots,\ \underline{y}_{p(p-3)/2},\ \underline{z}_1,\ \underline{z}_2,\ \ldots,\ \underline{z}_{p(p-1)(p-5)/6}] \tag{13}$$

for S_3.

Let us further, complete the basis given in (13) to another orthonormal basis

$$[(1/\sqrt{v})\, E_{v,1},\ \underline{x}_2,\ \underline{x}_3,\ \ldots,\ \underline{x}_p,\ \underline{y}_1,\ \underline{y}_2,\ \ldots,\ \underline{y}_{p(p-3)/2},\ \underline{z}_1,\ \underline{z}_2,\ \ldots,\ \underline{z}_{p(p-1)(p-5)/6},\ \underline{x}_{123(1)},\ \underline{x}_{123(2)},\ \ldots,\ \underline{x}_{123(5)},\ \underline{x}_{124(1)},\ \ldots,\ \underline{x}_{124(5)},\ \ldots] \tag{14}$$

for S_4 Let

$$\begin{aligned}
\underline{t}'\,\underline{x}_{ijk(1)} &= (t_{ijk} - t_{ikj}) + (1.2)^{\frac{1}{2}};\\
\underline{t}'\,\underline{x}_{ijk(2)} &= (t_{ijk} + t_{ikj} - 2t_{jik}) + (2.3)^{\frac{1}{2}};\\
\underline{t}'\,\underline{x}_{ijk(3)} &= (t_{ijk} + t_{ikj} + t_{jik} - 3t_{jki}) + (3.4)^{\frac{1}{2}};\\
\underline{t}'\,\underline{x}_{ijk(4)} &= (t_{ijk} + t_{ikj} + t_{jik} + t_{jki} - 4t_{kij}) + (4.5)^{\frac{1}{2}};\\
\underline{t}'\,\underline{x}_{ijk(5)} &= (t_{ijk} + t_{ikj} + t_{jik} + t_{jki} + t_{kij} - 5t_{kji}) + (5.6)^{\frac{1}{2}},
\end{aligned} \tag{15}$$

We define

$$A_2^* = \sum_{i=2}^{p} \underline{x}_i \underline{x}_i'$$

$$A_3^* = \sum_{s=1}^{p(p-3)/2} \underline{y}_s \underline{y}_s' ; \tag{16}$$

$$A_4^* = \sum_{i=1}^{p(p-1)(p-5)/6} \underline{z}_i' \underline{z}_i :$$

$$A_1^* = \sum_{i<j<k} \sum \sum \sum_{l=1}^{5} x_{ijk}(l)\, x_{ijk}'(l) ;$$

to get the analysis of the given MET(p) design.

A solution of the reduced normal equations

$$C\hat{\underline{t}} = \underline{Q} \tag{17}$$

will be

$$\hat{\underline{t}} = \left[\sum_{i=1}^{4} (1/\phi_i)\, A_i^* \right] \underline{Q} \tag{18}$$

Following Raghavarao (1962), we get

$$\hat{t} = (1/\phi_i)\, A_1^* + (1/\phi_2)\, A_2^* + (1/\phi_3)\, A_3^* + (1/\phi_4)$$

$$(I_v - A_1^* - A_2^* - A_3^*). \tag{19}$$

Now, A_3^* can be worked out as given below :

$$A_3^* = D\,[(1/6(p-4))\, I_v^* - ((p-2)/12(p-3)(p-4))\, A_1$$

$$- ((p-1)/9\overset{*}{v}(p-2)(p-4)\, E\overset{*}{v}, \overset{*}{v}]\, D' \tag{20}$$

$$- (1/v)\, Ev, v - A_2^* ,$$

where A_1 is an idempotent matrix defined by Aggarwal (1974) anc D' is the incidence matrix of the triangular design with parameters

$$v^* = p\,(p-1)/2, \qquad b^* = p\,(p-1)\,(p-2),$$

$$r^* = 6\,(p-2), \qquad k^* = 3;\ \lambda_1^* = 6,\ \lambda_2^* = 0. \tag{21}$$

Therefore, the solution of the normal equations given in (18) is given by

$$\hat{t}_{ijk} = [(Q_{i..}^{*} + Q_{j..}^{*} + Q_{k..}^{*})/6(p-2)(p-3)\phi_1$$

$$+ (Q_{ij.}^{*} + Q_{ik.}^{*} + Q_{jk.}^{*} - 2((Q_{i..}^{*} + Q_{j..}^{*} + Q_{k..}^{*})/$$

$$(p-2))/6(p-4)\phi_2 + (Q_{ijk} + (Q_{i..}^{*} + Q_{j..}^{*} + Q_{k..}^{*})/$$

$$(p-3)(p-4) - (Q_{ij.}^{*} + Q_{ik.}^{*} + Q_{jk.}^{*})/(p-4)$$

$$+ Q_{ijk}^{*})/6\phi_3 + (5Q_{ijk} - Q_{ijk}^{*})/6\phi_4], \qquad (22)$$

for $i, j, k = 1, 2, \ldots, p$.

The sum of squares (S.S.) for g.c.a., first order s.c.a., second order s.c.a. and reciprocal effects, will be $(1/\phi_2)\ \underline{Q}'\ A_2^{*}\ \underline{Q}$, $(1/\phi_3)\ \underline{Q}'\ A_3^{*}\underline{Q}$, $(1/\phi_4)\ \underline{Q}'\ A_4^{*}\ \underline{Q}$, $(1/\phi_4)\ \underline{Q}'\ [I_v - A_1^{*} - A_2^{*} - A_3^{*})\ \underline{Q}$ and $(1/\phi_1)\ \underline{Q}'\ A_1^{*}\ \underline{Q}$, respectively.

Now we shall work out

$$\underline{Q}'\ A_2^{*}\ \underline{Q} = \underline{Q}'\ \Sigma\ \underline{x}_i\ \underline{x}_i')\ \underline{Q} \qquad (23)$$

$$= \sum_{i=2}^{p}\left[\sum_{j=1}^{i-1} \underline{Q}_{j..} - (i-1)\underline{Q}_{i..}^{*}\right]^2 \Big/ [3i(i-1)(p-2)(p-3)]$$

$$= [1/12(p-2)(p-3)]\,[(Q_{2..}^{*} - Q_{2..}^{*})^2/2.1$$

$$+ (Q_{1..}^{*} + Q_{2..}^{*} - 2Q_{3..}^{*})^2/3.2$$

$$+ (Q_{1..}^{*} + Q_{2..}^{*} + Q_{3..}^{*} - 3Q_{4..}^{*})^2/4.3 + \ldots +$$

$$(Q_{1..}^{*} + Q_{2..}^{*} + \ldots + \ldots - (p-1)\,Q_{p..}^{*})^2/p(p-1)]$$

$$= [1/12(p-2)(p-3)]\left[\sum_{i=1}^{p} Q_{i..}^{*2}(p-1)/p + (1/p)\sum_{i=1}^{p} Q_{i..}^{*2}\right]$$

$$= [1/12(p-2)(p-3)]\left[\sum_{i=1}^{p} Q_{i..}^{*2}\right]$$

$$= \sum_{i=1}^{p} Q_{i\cdot\cdot}^{*2} / 12\,(p-2)\,(p-3)\,. \tag{24}$$

The above result can be put in the form of theorem given below :

THEOREM. The sum of squares (S.S.) for g.c.a. effects are given by

$$\underline{Q}'\, A_1^*\, \underline{Q} = \sum_{i=1}^{p} Q_{i\cdot\cdot}^{*2} / 12\,(p-2)\,(p-3). \tag{25}$$

Similarly S.S. for first order s.c.a., second order s. c. a. and reciprocal effects can be worked out. The various S.S.'s are given in Table 1.

Illustration Let us construct the series of Modified Extended Triangular $MET(p)$ design with the parameters

$$v = p\,(p-1)\,(p-2),\ b = p\,(p-1)/2,\quad r = 3,$$

$$k = 6\,(p-2),\ \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = 0,\quad \lambda_4 = 0. \tag{26}$$

when $p = 6$, we have

$$v = 120,\quad b = 15,\quad r = 3$$

$$k = 24,\ \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = 0,\ \lambda_4 = 0 \tag{27}$$

First arrange $v = p(p-1)\,(p-2)/6 = 20$ (for $p = 6$) treatments in 6 square arrays $A_1, A_2, \ldots, A_6$ each of order 5 as the following :

$A_1 =$

×	123	124	125	126
123	×	134	135	136
124	134	×	145	146
125	135	145	×	156
126	136	146	156	×

$A_2 =$

×	123	124	125	126
123	×	234	235	236
124	234	×	245	246
125	235	245	×	256
126	236	246	256	×

$A_3 =$

×	123	134	135	136
123	×	234	235	236
134	234	×	345	346
135	235	345	×	356
136	236	356	356	×

(28)

TABLE 1—ANOVA TABLE

Source	*d.f.*	*S. S.*	*M. S.*
Blocks ignoring treatments	$(b-1)$	$\sum_l B_l^2/k - C.F.$	—
g.c.a. effects eliminating blocks	$(p-1)$	$\sum_i Q_{i..}^{*2}/12(p-2)(p-3)\phi_2$	M_g
First order s.c.a. effects eliminating blocks	$p(p-3)/2$	$[\sum_{i<j}\sum Q_{ij.}^{*2} - (\sum_i Q_{i..}^{*2})/(p-2)]/6(p-4)\phi_8$	M_s
Second order s.c.a. effects eliminating blocks	$p(p-1)(p-5)/6$	$[5\sum_{i<j<k}\sum\sum Q_{ijk}^{*2} - (\sum_{i<j}\sum Q_{ij.}^{*2} - \sum_i Q_{i..}^{*2}/2(p-3))/(p-4)]/6\phi_4$	M_{ss}
Reciprocal effects	$5p(p-1)(p-2)/6$	$\sum_{i<j<k}\sum\sum Q_{jik}^{*2}/6\phi_1$	M_{rr}
Error	$vr-b-v+1$	By subtraction	M_e
Total	$vr-1$	$\sum_i\sum_j\sum_k\sum_l Y_{ijkl}^2 - C.F.$	

$$A_4 = \begin{matrix} \times & 124 & 134 & 145 & 146 \\ 124 & \times & 234 & 245 & 246 \\ 134 & 234 & \times & 345 & 346 \\ 145 & 245 & 345 & \times & 456 \\ 146 & 246 & 346 & 456 & \times \end{matrix}$$

$$A_5 = \begin{matrix} \times & 125 & 135 & 145 & 156 \\ 125 & \times & 235 & 245 & 256 \\ 135 & 235 & \times & 345 & 356 \\ 145 & 245 & 345 & \times & 456 \\ 156 & 256 & 356 & 456 & \times \end{matrix}$$

$$A_6 = \begin{matrix} \times & 126 & 136 & 146 & 156 \\ 126 & \times & 236 & 246 & 256 \\ 136 & 236 & \times & 346 & 356 \\ 146 & 246 & 346 & \times & 456 \\ 156 & 256 & 356 & 456 & \times \end{matrix}$$

By mapping each treatment ijk of the extended tringular design (28), we obtain the MET(6) design with the parameters as given in (27).

The Q^*_{ijk}'s and the Q_{ijh}'s the adjusted treatment totals for MET(6) are given in Table 2 and Table 3 respectively. The various sum of squares (S.S.) due to the different effects have been computed with the help of the formulae shown in Table 1 and are given in ANOVA Table 4.

TABLE 2—Q^*_{ijk}—ADJUSTED TREATMENT TOTALS

Triallel Cross combinations	*Adjusted treatment total*	*Triallel cross combinations*	*Adjusted treatment totals*
Q^*_{123}	— 93.00	Q^*_{234}	—20.75
Q^*_{124}	— 92.25	Q^*_{235}	—27.75
Q^*_{125}	— 57.75	Q^*_{236}	— 9.25
Q^*_{126}	— 57.00	Q^*_{245}	0 50
Q^*_{134}	— 57.00	Q^*_{246}	38.75
Q^*_{135}	— 40.00	Q^*_{256}	61.00
Q^*_{136}	— 30.25	Q^*_{345}	56.50
Q^*_{145}	— 14.00	Q^*_{346}	54.75
Q^*_{146}	15.50	Q^*_{356}	90.50
Q^*_{156}	47.25	Q^*_{456}	134.25

TABLE—$3Q_{ijk}$ – ADJUSTED TREATMENT TOTALS

− 30.17 (123)	− 18.17 (132)	− 29.17 (213)	− 21.17 (231)	1.83 (312)	3.83 (321)
− 27.71 (124)	− 25.71 (142)	− 29.71 (214)	− 22.71 (241)	4.29 (412)	9.29 (321)
− 27.12 (125)	− 28.12 (152)	− 25.12 (215)	− 18.12 (251)	13.88 (512)	26.88 (521)
− 34.67 (126)	− 34.67 (162)	− 32.67 (216)	− 24.67 (261)	30.33 (612)	31.33 (621)
− 64.02 (146)	− 62.42 (164)	− 8.42 (416)	8.58 (461)	70.51 (614)	61.58 (641)
− 67.87 (156)	− 56.87 (165)	24.13 (516)	24.13 (561)	64.13 (615)	59.13 (651)
− 26.28 (243)	24.29 (243)	− 8.29 (324]	− 10.29 (342)	21.71 (423)	26.71 (432)
− 42.13 (235)	− 30.13 (253)	− 13.13 (325)	− 15.13 (352)	30.87 (523)	41.87 (532)
− 29.67 (134)	− 34.67 (143)	− 20.67 (314)	− 10.67 (241)	15.33 (413)	2[illegible].33 (431)
− 40.50 (135)	− 34.50 (153)	− 20.50 (215)	− 10.50 (351)	29.50 (513)	36.50 (531)
− 51.71 (136)	− 41.71 (163)	− 22.71 (316)	− 13.71 (361)	51.29 (613)	55.29 (631
− 60.50 (145)	− 48.50 (154)	− 4.50 (415)	− 4.50 (451)	45.50 (514)	40.50 (541)
− 39.04 (236)	− 35.04 (263)	− 23.04 (326)	− 18.04 (362)	52.96 (623)	52.96 (632)
− 37.08 (245)	− 40.08 (254)	− 0.08 (425)	− 6.08 (452)	46.92 (524)	36.92 (542)
− 43.71 (246)	− 34.71 (264)	− 4.71 (426)	11.29 (462)	51.29 (624)	59.29 (642)
− 56.17 (256)	− 48.17 (265)	16.83 (526)	26.83 (562)	68.83 (625)	52.83 (652)
− 26.08 (345)	− 16.08 (354)	5.92 (435)	5.92 (453)	44.92 (534)	41.92 (543)
− 25.38 (346)	− 35.38 (364)	− 4.38 (436)	3.62 (463)	56.62 (634)	59.62 (643)
− 34.25 (356)	− 38.25 (365)	24.75 (536)	21.75 (563)	60.75 (635)	55.75 (653)
1.63 (456)	− 10.70 (465)	24.63 (546)	20.63 (564)	49.63 (645)	49.63 (654)

Note : Number within the parentheses indicate the respective trialled cross combination.

TABLE 4—ANOVA TABLE

Source	*d.f.*	*Some of squores*	*Mean Squares*
Blocks ignoring treatments	14	11527.05	823.36
g. c. a. effects eliminating blocks	5	956.39	191.28
First order s.c.a. effects eliminating blocks	9	17444.64	1938.29
Second order s.c.a. effects eliminating blocks	5	12216.59	2443.32
Reciprocal effects	100	6333.41	63.33
Error	226	21444.18	94.89
Total	359	69922.26	

The estimates of the various genetic effects, variances of these estimates and variances of the elementary contrasts of these estimates, are

$$\hat{g}_i = Q^*_{i..}/6(p-2)(p-3)\phi_1;$$

$$\hat{s}_{ij} = [Q^*_{ij} - 6(p-3)(\hat{g}_i + \hat{g}_j)]/6(p-4)\phi_2;$$

$$\hat{s}_{ijk} = [1/6Q^*_{ijk} - (\hat{g}_i + \hat{g}_j + \hat{g}_k) - (\hat{s}_{ij} + \hat{s}_{ik} + \hat{s}_{jk})]/\phi_3$$

$$\hat{r}_{ijk} = Q_{ijk} - Q^*_{ijk}/6]/\phi_4;$$

$$V(\hat{g}_i) = [(p-1)\sigma^2]/[3p(p-2)(p-3)\phi_1];$$

$$V(\hat{g}_i - \hat{g}_j) = 2\sigma^2/[3(p-2)(p-3)\phi_1], \; i \neq j$$

$$V(\hat{s}_{ij} - \hat{s}_{ik}) = (p-3)\sigma^2/[3(p-2)(p-4)\varphi_2], \; i \neq j, k; \; j \neq k$$

$$V(\hat{s}_{ij} - \hat{s}_{kl}) = \sigma^2/[3(p-2)\varphi_2], \; i \neq j, k, l; \; j \neq k, l; \; k \neq l;$$

$$V(\hat{s}_{ijk} - \hat{s}_{ijl}) = (p-5)\sigma^2/[3(p-3)\phi_3], \; i \neq j, k, l; \; j \neq k, l; \; k \neq l;$$

$$V(\hat{s}_{ijk} - \hat{s}_{ilm}) = (p-5)(p^2 - 6p + 3)\sigma^2/[3(p-3)(p-4)^2\phi_3],$$
$$i \neq j, k, l, m\ ;\ j \neq k, l, m\ ;\ k \neq l, m\ ;\ l \neq m;$$

$$V(\hat{s}_{ijk} - \hat{s}_{mn}) = (p^2 - 10p + 27)\sigma^2/[3(p-3)(p-4)\phi_3],$$
$$i \neq j, k, l, m, n;\ j \neq k, l, m, n;\ k \neq l, m, n;$$
$$l \neq m, n\ ;\ m \neq n$$

$$V(\hat{r}_{ijk}) = (5/6)\sigma^2/\phi_4,\ i \neq j, k;\ j \neq k;$$

$$V(\hat{r}_{ijk} - \hat{r}_{ijl}) = (5/3)\sigma^2/\phi_4,\ i \neq j, k;\ j \neq k;$$

$$V(\hat{r}_{ijk} - \hat{r}_{ilm}) = (5/3)\sigma^2/\phi_4,\ i \neq j, k;\ j \neq k.$$

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